

Partial rip scenario - a cosmology with a growing cosmological term

H. Štefančić*

Theoretical Physics Division, Rudjer Bošković Institute,
P.O.Box 180, HR-10002 Zagreb, Croatia

Abstract

A cosmology with the growing cosmological term is considered. If there is no exchange of energy between vacuum and matter components, the requirement of general covariance implies the time dependence of the gravitational constant G . Irrespectively of the exact functional form of the cosmological term growth, the universe ends in a de Sitter regime with a constant asymptotic Λ , but vanishing G . Although there is no divergence of the scale factor in finite time, such as in the “Big Rip” scenario, gravitationally bound systems eventually become unbound. In the case of systems bound by non-gravitational forces, there is no unbounding effect, as the asymptotic Λ is insufficiently large to disturb these systems.

Cosmological observations of increasing quantity, quality and diversity have established a new picture of the universe, its composition and dynamics [1]. Measurements of luminosity-redshift relations for the supernovae of type Ia (SNIa) [2] and the temperature anisotropies of the cosmic microwave background radiation (CMBR) [3], in agreement with other cosmological observations, have determined

*shrvoje@thphys.irb.hr

that the universe is presently in the phase of accelerated expansion, which is attributed to a new component of the universe named *dark energy*. The origin of dark energy (which together with *dark matter* constitutes the “dark sector” of the universe) has not yet been clarified. Pretenders to the title of dark energy are, however, numerous. The first in a long line of candidates is certainly the cosmological constant (CC), also known as cosmological term [4, 5, 6]. This conceptually simple choice, which naturally fits into the formalism of general relativity (GR), is, however, burdened with problems which, to be solved, require a highly unnatural amount of fine-tuning. Namely, quantum field theory contributions to the vacuum energy (which is equivalent to the CC) and the observed value of the CC differ by many orders of magnitude. Furthermore, there is a coincidence problem of understanding why the CC energy density and the non-relativistic matter energy density are comparable at the present epoch. These problems were a strong incentive towards the development of dynamical dark energy models. One of the most studied classes of such models are *quintessence* models [7]. Quintessential models describe dark energy in terms of the scalar field slowly rolling in a potential. Another appealing proposal is the Chaplygin gas [8]. The most interesting feature of this model is the possibility of unified description of the “dark sector”, i.e. the unification of dark matter and dark energy. Another alternative to quintessence with the similar properties of effective unification of dark sector is Cardassian cosmology [9]. Other interesting models include the non-perturbative effects of vacuum energy [10] and the fluctuating CC [11]. The consideration of the CC as a dynamical quantity in the framework of the renormalization group equation [12, 13, 14] is another promising approach.

A large majority of dark energy models describes dark energy in terms of the equation of state (EOS)

$$p_d = w \rho_d, \quad (1)$$

where w is the parameter of the EOS, while p_d and ρ_d denote the pressure and the energy density of dark energy, respectively. The value $w = -1$ is characteristic of the cosmological constant, while the dynamical models of dark energy generally have $w \geq -1$. A recent examination of the dark energy EOS, based on the data from CMBR, SNIa, large scale structure (LSS) and Hubble parameter measurements from the Hubble Space Telescope (HST), assuming the redshift independent parameter w , restrict w to be in the interval $-1.38 < w < -0.82$ at the 95% confidence level [15]. This result raises a question about the possibility of dark energy models with the supernegative EOS, i.e. with $w < -1$. This new sort of dark energy, first analyzed in [16] which was followed by numerous analyses [17], has soon deserved a term of its own, *phantom energy*. The most interesting feature of phantom energy models is the possibility of a divergence of the scale factor of the universe a in finite time. Such a behaviour of the scale factor has a dramatic effect on all bound systems. Namely, the bound systems become unbound at some finite time interval before the onset of the divergence in a . This type of the fate of the universe is known as a “Big Rip” scenario [18].

In this letter we consider the case of the growing cosmological term Λ and its implications for the asymptotic expansion of the universe and the destiny of the bound systems. Let us start with the specification of the main characteristics of

the components of the universe. We assume that we have two components of the universe: non-relativistic matter and the variable cosmological term. The inclusion of radiation or other components is straightforward, but without any substantial influence on the future evolution of the universe. Non-relativistic matter has the EOS

$$p_m = \gamma \rho_m, \quad (2)$$

where $\gamma \geq 0$ is the parameter of the EOS and p_m and ρ_m represent the pressure and the energy density of non-relativistic matter, respectively. The conservation of the energy-momentum tensor of non-relativistic matter, $T_{m;\nu}^{\mu\nu} = 0$, leads to the standard relation of the evolution of ρ_m with the scale factor

$$\rho_m = \rho_{m,0} \left(\frac{a}{a_0} \right)^{-3(1+\gamma)}. \quad (3)$$

The equation of state of the cosmological term is

$$p_\Lambda = -\rho_\Lambda, \quad (4)$$

where p_Λ is the pressure and ρ_Λ is the energy density of the cosmological term. The requirement of the conservation of the energy-momentum tensor of cosmological term, $T_\Lambda^{\mu\nu} = \rho_\Lambda g^{\mu\nu}$, leads to a constant cosmological term energy density ρ_Λ . If we, however, consider a variable (time-dependent) ρ_Λ , $T_\Lambda^{\mu\nu}$ can no longer be conserved. Models with variable $\Lambda \equiv 8\pi G \rho_\Lambda$ and G were extensively studied in [19]. The consideration of phantom energy models in an analogous framework leads to an interesting general finding on the future evolution of the universe [20]. In the setting described above, the general covariance of the Einstein equation

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}, \quad (5)$$

where $G_{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu} = T_m^{\mu\nu} + T_\Lambda^{\mu\nu}$, can be maintained if the gravitational constant G acquires space-time dependence. This fact can be interpreted as a modification of the dynamics of General Relativity. We effectively describe this additional dynamics by promoting G into a function of space-time coordinates. We further assume that G depends on time only, i.e. $G = G(t)$ ¹. The condition $(G(t)T^{\mu\nu})_{;\nu} = 0$ can be expressed as

$$d(G(\rho_m + \rho_\Lambda)a^3) = -G(p_m + p_\Lambda)da^3, \quad (6)$$

which gives the law of evolution of $G(t)$

$$\dot{G}(\rho_m + \rho_\Lambda) + G\dot{\rho}_\Lambda = 0. \quad (7)$$

Apart from the procedure displayed above, there exist other ways how the variability of the cosmological term can be incorporated into the formalism of General Relativity. One possibility is to allow the interchange of energy and momentum between the cosmological term (vacuum) and matter and radiation components

¹Interesting models with variable G are also obtained in the formalism of Quantum field theory in the curved space-time [21].

[22]. In this model, the energy-momentum tensors of separate components (non-relativistic matter or radiation and vacuum energy) are not conserved, but the total energy-momentum tensor is. Furthermore, G is time-independent in this setting. It is interesting to see that both the model of our paper and the model given in [22] follow from equation (6) under different assumptions. The dynamics of our model (7) is obtained assuming that the vacuum energy is time dependent and that the tensor of energy-momentum of the matter component is separately conserved. The dynamics of [22], expressed by their equation (5a), is recovered under the assumption that G is time-independent and that there exists an exchange of energy between the vacuum and the matter components.

The set of equations governing the evolution of the universe is completed by the Friedmann equations for the scale factor

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G(\rho_m + \rho_\Lambda), \quad (8)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho_m + \rho_\Lambda + 3p_m + 3p_\Lambda). \quad (9)$$

Given the set of equations (3), (7), (8) and (9), we can investigate the future evolution of the universe for a general growing ρ_Λ . The law of evolution of ρ_m (3) clearly shows that at sufficiently distant future times we have $\rho_\Lambda \gg \rho_m$. In this limit, equations (7) and (8) become

$$\dot{G}\rho_\Lambda + G\dot{\rho}_\Lambda = 0, \quad (10)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho_\Lambda. \quad (11)$$

From (10) it follows that at distant times we have $G\rho_\Lambda = \text{const}$. Therefore, in equation (11) we can disregard the k/a^2 term at sufficiently distant times. We finally obtain the equations governing the evolution of the universe

$$\frac{d(G\rho_\Lambda)}{dt} = 0, \quad (12)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho_\Lambda. \quad (13)$$

The solution of the equation displayed above is of the form

$$a \sim e^{\sqrt{\frac{\Lambda_\infty}{3}}t}, \quad (14)$$

where $\Lambda_\infty = 8\pi(G\rho_\Lambda)_{t \rightarrow \infty}$. In the distant future the universe enters the de Sitter regime of expansion. This fact does not depend on the concrete form of ρ_Λ , as long as it is a growing function of time (explicitly or implicitly via some other quantity depending on time, such as the scale factor a). Furthermore, the growth of ρ_Λ implies that G is a decreasing function at distant times. The unbounded growth of ρ_Λ results in vanishingly small values of G at distant times.

Let us further examine the complete evolution of the universe for a class of growing ρ_Λ models. We assume the following form for the variable cosmological term energy density:

$$\rho_\Lambda = \rho_{\Lambda,0} \left(\frac{a}{a_0}\right)^{-3(1+\eta)}, \quad (15)$$

where $\eta < -1$. We consider the flat universe case, $k = 0$, which implies that the total energy density ρ_0 equals the critical energy density $\rho_{c,0}$, and introduce a dimensionless parameter $\Omega_\Lambda^0 \equiv \rho_{\Lambda,0}/\rho_0$. The subscript or superscript 0 denotes the present time throughout the paper. What remains is solving equations (7) and (8) given the laws of evolution of the energy densities (3) and (15). The quantity G evolves according to the law

$$G = G_0 \left(\Omega_\Lambda^0 \left(\frac{a}{a_0} \right)^{-3(\eta-\gamma)} + 1 - \Omega_\Lambda^0 \right)^{-\frac{1+\eta}{\eta-\gamma}}, \quad (16)$$

while the scale factor is implicitly given by the expression

$$H_0(t - t_0) = \int_1^{a/a_0} x^{\frac{1}{2}(1+3\gamma)} \left(\Omega_\Lambda^0 x^{-3(\eta-\gamma)} + 1 - \Omega_\Lambda^0 \right)^{\frac{1}{2}\frac{1+\gamma}{\eta-\gamma}} dx. \quad (17)$$

The evolution of the scale factor of the universe a with time is shown in figure 1. The dependences of a on time for different values of the parameter η differ from each other the most in the distant past and the distant future. Graphs of the evolution of the scale factor in the distant future also reveal the beginning of the exponential expansion, i.e. the onset of the de Sitter regime. The time dependence of $G(t)$, depicted in figure 2, is more sensitive to the value of the parameter η . Figure 2 displays some general features of the dynamics of $G(t)$. In the early universe (for small values of the scale factor) the change of $G(t)$ is very slow. The rate of change is the greatest around the present epoch, while at large times $G(t)$ tends towards 0. The pronounced dynamics of $G(t)$ for more negative values of the parameter η would probably be the best testing ground for the proposed class of models, i.e. it would provide the most stringent constraint on the growth of the cosmological term energy density with the scale factor, described by the parameter η . The time evolution of Λ is displayed in figure 3. In the early universe the value of Λ approaches 0 for $\eta < -1$. For large times, the function $\Lambda(t)$ tends to its asymptotic value Λ_∞ . For more negative values of η , the asymptotic value Λ_∞ increases and it is approached slower. The dependence of the asymptotic value Λ_∞ on the parameter η is shown in figure 4. For the chosen range of the parameter η , approximately consistent with the constraint of reference [15], the quantity Λ_∞ grows quite modestly, definitely remaining of the same order of magnitude.

The evolutions of $\Lambda(t)$ and $G(t)$ depicted in figures 2 and 3, respectively, show that the universe ends up in a state with finite Λ_∞ and vanishing G_∞ . An important question for such a cosmology is the destiny of bound systems. First, we focus on gravitationally bound systems. Let us consider a spherically symmetric system with a mass M at the centre, in the cosmology with $\Lambda_\infty \approx \text{const}$ and $G_\infty \approx 0$. We can treat this system as approximately static. The metrics of such a system is then given by [23]

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\Omega^2, \quad (18)$$

where

$$e^\nu = e^{-\lambda} = 1 - \frac{2G_\infty M}{r} - \frac{1}{3}\Lambda_\infty r^2. \quad (19)$$

In the Newtonian limit, the gravitational potential becomes

$$\phi(r) = -\frac{G_\infty M}{r} - \frac{1}{6}\Lambda_\infty r^2. \quad (20)$$

As G_∞ is vanishingly small and Λ_∞ is finite, presently gravitationally bound systems cannot remain bound. The unbounding happens at some instance before a state with $\Lambda_\infty \approx \text{const}$ and $G_\infty \approx 0$ is achieved. Clusters of galaxies, galaxies, stars and stellar systems will fall apart. Planets will lose their atmospheres. The gravitational interaction will become fully dominated by the asymptotic value of the cosmological term.

Next, we turn to systems bound by forces other than gravitational, e.g. strong or electromagnetic. In these systems, G_∞ is clearly of no relevance. The dependence of Λ_∞ on the parameter η is shown in figure 4. For values of η not much smaller than -1 , Λ_∞ remains of the same order of magnitude as Λ_0 . As Λ_0 obviously has no unbounding effect on non-gravitationally bound systems, and given the difference of many orders of magnitude between typical interaction scales of gravitational and non-gravitational forces, neither Λ_∞ will be able to unbound non-gravitationally bound systems. The break-up of atoms and nucleons induced by the variable cosmological term will not happen.

Finally, let us discuss constraints on the model parameters. Cosmological observations impose rather strong constraints on the time dependence of the gravitational constant G . The observational bounds on the variation of G differ in their origin and refer to different epochs of the universe expansion [24, 25]. The observations of spin-down rate of pulsars yield constraints $|\dot{G}/G| \leq (2.2 - 5.5) \times 10^{-11} \text{ yr}^{-1}$ for the pulsar PSR B0655+64 [26], and $|\dot{G}/G| \leq (1.4 - 3.2) \times 10^{-11} \text{ yr}^{-1}$ for the pulsar PSR J2019+2425 [25, 27], where the range in constraints comes from the uncertainties in the neutron star equation of state. The effects of the variation of G on the orbital period of the neutron star-white dwarf binary system PSR B1855+09 give the constraint $\dot{G}/G = (-1.3 \pm 2.7) \times 10^{-11} \text{ yr}^{-1}$ [27, 28]. The consideration of effects of the variation of G on the Chandrasekhar mass in double-neutron-star binaries yields a constraint of $\dot{G}/G = (-0.6 \pm 4.2) \times 10^{-12} \text{ yr}^{-1}$ [29]. The observations of SNIa give $\dot{G}/G \leq 10^{-11} \text{ yr}^{-1}$ at redshifts $z \sim 0.5$ [30], while the Big Bang Nucleosynthesis constrains the gravitational constant during BBN to be $G_{BBN}/G_0 = 1.01^{+0.20}_{-0.16}$ at the 68.3 % confidence level [31]. One of the strongest constraints on the variation of G comes from the Lunar Laser Ranging (LLR) and amounts to $\dot{G}/G = (0.46 \pm 1.0) \times 10^{-12} \text{ yr}^{-1}$ [32]. Figure 2 shows that constraints on \dot{G}/G from the distant past are more easily satisfied than those from the recent past since the variation of G is stronger in the recent than in the distant past. Namely, the rate of change \dot{G} grows with time in absolute value from the distant past to the present epoch, while the value of G decreases with time. For given η , of all past epochs, the quantity \dot{G}/G has the strongest variation for the present epoch. On the other hand, the strongest observational limit on \dot{G}/G comes from LLR [32] at the present epoch (with $z = 0$). Therefore, the comparison of theoretical expressions and observational bounds at the present epoch can produce the most stringent constraints on the parameter η . From equations (7) and (15) the theoretical expression for the rate \dot{G}/G at $z = 0$ becomes

$$\eta = -1 + \frac{1}{3\Omega_\Lambda^0 H_0} \left(\frac{\dot{G}}{G} \right)_0. \quad (21)$$

Using the value $H_0 = 72 \text{ km/s/Mpc}$ for the Hubble parameter, $\Omega_\Lambda^0 = 0.7$ and the

values for \dot{G}/G from [32], we obtain the following constraints on the parameter η : $\eta \geq -1.0035$ at the 1σ level, $\eta \geq -1.01$ at the 2σ level and $\eta \geq -1.016$ at the 3σ level. Other observational constraints yield weaker limits on η . The constraints from BBN [31], e.g., using (16) yield $\eta \geq -1.1881$ at the 68.3 % confidence level, which is a much weaker limit. All the figures are plotted using a broader range of parameter η than the range constrained by LLR to better illustrate the form of dependencies of functions a , G and Λ on time and η . The obtained constraints result in strong limits on the parameter η which must be close to -1. However, as long as the condition $\eta < -1$ is satisfied, no matter how close η is to -1, the main conclusions of this paper remain unaltered. In another words, even for very slow growth of ρ_Λ (which satisfies all the conditions on the variation of G) the partial rip scenario is valid, i.e., in the distant future the gravitationally bound systems become unbound, while the nongravitationally bound systems remain bound.

Considerations presented above show that a universe with the growing cosmological term and the time-dependent G has very interesting properties. The energy density of dark energy component grows with the scale factor (like in phantom energy models), but the parameter of the EOS remains -1 (unlike in phantom energy models where the same parameter is less than -1). The universe ends up in a de Sitter regime with constant Λ_∞ and vanishing G_∞ . The expansion continues infinitely, without any divergence of the scale factor in finite time. However, despite the absence of the abrupt ending of the universe in a “Big Rip” event, accompanied by the unbounded growth of dark energy density, all bound systems do not remain bound. Gravitationally bound systems become unbound in the distant future in an interplay of constant repulsion of Λ_∞ and vanishing attraction mediated by G_∞ . Systems bound by non-gravitational forces, however, do not share the destiny of their gravitationally bound counterparts. Numerical calculations show that, for a reasonable range of the parameter η , the value Λ_∞ is not sufficiently large to disturb systems bound by, e.g. strong or electromagnetic forces. It is important to emphasize once more that all these findings are not dependent on the size of η , as long as it remains less than -1. For the case of η close to -1, all the effects described above happen, only the variations of $G(t)$ and $\Lambda(t)$ are much milder and the onset of the saturation of $\Lambda(t)$ in the future happens much later.

Models with the growing cosmological term energy density and the time-dependent G have testable predictions about the past evolution of the universe. The confrontation of these predictions with the host of cosmological observations available today gives constraints on the parameters of the model, above all on the parameter η . These constraints determine the destiny of the universe and its structures and how far in the future the possible dramatic unbounding effects lie in front of us.

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References

- [1] W.L. Freedman, M.S. Turner, *Rev. Mod. Phys.* 75 (2003) 1433; S.M. Carroll, [astro-ph/0310342](#).
- [2] A.G. Riess et al., *Astron. J.* 116 (1998) 1009; S. Perlmutter et al., *Astrophys. J.* 517 (1999) 565.
- [3] P. de Bernardis et al., *Nature* 404 (2000) 955; A.D. Miller et al. *Astrophys. J. Lett.* 524 (1999) L1; S. Hanany et al., *Astrophys. J. Lett.* 545 (2000) L5; N.W. Halverson et al., *Astrophys. J.* 568 (2002) 38; B.S. Mason et al., *Astrophys. J.* 591 (2003) 540; D.N. Spergel et al., *Astrophys. J. Suppl.* 148 (2003) 175; L. Page et al. [astro-ph/0302220](#).
- [4] S. Weinberg, *Rev. Mod. Phys.* 61 (1989) 1.
- [5] P.J.E. Peebles, B. Ratra, *Rev. Mod. Phys.* 75 (2003) 559.
- [6] T. Padmanabhan, *Phys. Rept.* 380 (2003) 235.
- [7] B. Ratra, P.J.E. Peebles, *Phys. Rev. D* 37 (1988) 3406; P.J.E. Peebles, B. Ratra, *Astrophys. J.* 325 (1988) L17; C. Wetterich, *Nucl. Phys. B* 302 (1988) 668; R.R. Caldwell, R. Dave, P.J. Steinhardt, *Phys. Rev. Lett.* 80 (1998) 1582; I. Zlatev, L. Wang, P.J. Steinhardt, *Phys Rev. Lett.* 82 (1999) 896.
- [8] A. Yu. Kamenshchik, U. Moschella, V. Pasquier, *Phys. Lett. B* 511 (2001) 265; N. Bilić, G.B. Tupper, R.D. Viollier, *Phys. Lett. B* 535 (2002) 17; M.C. Bento, O. Bertolami, A.A. Sen, *Phys. Rev. D* 66 (2002) 043507.
- [9] K. Freese, M. Lewis, *Phys. Lett. B* 540 (2002) 1; K. Freese, *Nucl. Phys. B (Proc. Suppl.)* 124 (2003) 50.
- [10] L. Parker, A. Raval, *Phys. Rev. D* 60 (1999) 063512; Erratum-*ibid. D*67 (2003) 029901.
- [11] M. Ahmed, S. Dodelson, P.B. Greene, R. Sorkin, [astro-ph/0209274](#).
- [12] A. Babić, B. Guberina, R. Horvat, H. Štefančić, *Phys. Rev. D* 65 (2002) 085002; B. Guberina, R. Horvat, H. Štefančić, *Phys. Rev. D* 67 (2003) 083001.
- [13] I.L. Shapiro, J. Sola, *Phys. Lett. B* 475 (2000) 236; I.L. Shapiro, J. Sola, *JHEP* 0202 (2002) 006; I.L. Shapiro, J. Sola, C. Espana-Bonet, P. Ruiz-Lapuente, *Phys. Lett. B* 574 (2003) 149.
- [14] A. Bonanno, M. Reuter, *Phys. Lett. B* 527 (2002) 9; E. Bentivegna, A. Bonanno, M. Reuter, *JCAP* 0401 (2004) 001.
- [15] A. Melchiorri, L. Mersini, C.J. Ödman, M. Trodden, *Phys. Rev. D* 68 (2003) 043509.
- [16] R.R. Caldwell, *Phys. Lett. B* 545 (2002) 23.
- [17] C. Armendáriz-Picón, T. Damour, V. Mukhanov, *Phys. Lett. B* 458 (1999) 209; T. Chiba, T. Okabe, M. Yamaguchi, *Phys. Rev. D* 62 (2000) 023511; V. Faraoni, *Int. J. Mod. Phys. D* 11 (2002) 471; S.M. Carroll, M. Hoffman, M. Trodden, *Phys. Rev. D* 68 (2003) 023509; S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 562 (2003) 147; S. Nojiri, S.D. Odintsov, *Phys. Lett. B* 565 (2003) 1;

P. Singh, M. Sami, N. Dadhich, Phys. Rev. D 68 (2003) 023522; G.W. Gibbons, hep-th/0302199; L.P.Chimento, R. Lazkoz, Phys. Rev. Lett. 91 (2003) 211301; J.G. Hao, X.Z. Li, Phys. Rev. D 68 (2003) 083514; M.P.Dabrowski, T. Stachowiak, M. Szydlowski, Phys.Rev. D68 (2003) 103519; J.G. Hao, X.Z. Li Phys. Rev. D 67 (2003) 107303; J.G. Hao, X.Z. Li, astro-ph/0309746.

[18] R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, Phys. Rev. Lett. 91 (2003) 071301.

[19] A. Beesham, Nuovo Cimento 96 B (1986) 17; A. Beesham, Int. J. Theor. Phys. 25 (1986) 1295; A-M.M. Abdel-Rahman, Gen. Rel. Grav. 22 (1990) 655; M.S. Berman, Phys. Rev. D 43 (1991) 1075; M.S. Berman, Gen. Rel. Grav. 23 (1991) 465; R.F. Sistero, Gen. Rel. Grav. 32 (1991) 1265; D. Kalligas, P. Wesson, C.W.F. Everitt, Gen. Rel. Grav. 24 (1992) 351; T. Singh, A. Beesham, Gen. Rel. Grav. 32 (2000) 607; A.I. Arbab, A. Beesham, Gen. Rel. Grav. 32 (2000) 615; A.I. Arbab, Spacetime and Substance 1(6) (2001) 39; A.I. Arbab, astro-ph/0308068; J. Ponce de Leon, Class. Quant. Grav. 20 (2003) 5321.

[20] H. Štefančić, astro-ph/0310904.

[21] E. Elizalde, S.D. Odintsov, Phys. Lett. B 303 (1993) 240; E. Elizalde, S.D. Odintsov, Phys. Lett. B 321 (1994) 199; E. Elizalde, S.D. Odintsov, I.L. Shapiro, Class. Quant. Grav. 11 (1994) 1607; E. Elizalde, C. Lousto, S.D. Odintsov, A. Romeo, Phys. Rev. D 52 (1995) 2202.

[22] K. Freese, F.C. Adams, J.A. Frieman, E. Mottola, Nucl. Phys. B 287 (1987) 797.

[23] see e.g. R. Adler, M. Bazin, M. Schiffer, Introduction to general relativity, McGraw-Hill, 1975.

[24] C.M. Will, Living Rev. Rel. 4 (2001) 4.

[25] I.H. Stairs, Living Rev. Rel. 6 (2003) 5.

[26] I. Goldman, Mon. Not. Roy. Astr. Soc. 244 (1990) 184.

[27] Z. Arzoumanian, PhD thesis, Princeton University, Princeton, N.J., U.S.A., 1995.

[28] V.M. Kaspi, J.H. Taylor, M. Ryba, Astrophys. J. 428 (1994) 713.

[29] S.E. Thorsett, Phys. Rev. Lett. 77 (1996) 1432.

[30] E. Gaztanaga, E. Garcia-Berro, J.Isern, E. Bravo, I. Dominguez, Phys. Rev. D 65 (2002) 023506.

[31] C.J. Copi, A.N. Davis, L.M. Krauss, astro-ph/0311334.

[32] S.G. Turyshev, J.G. Williams, K. Nordtvedt Jr., M. Shao, T.W. Murphy Jr., gr-qc/0311039.

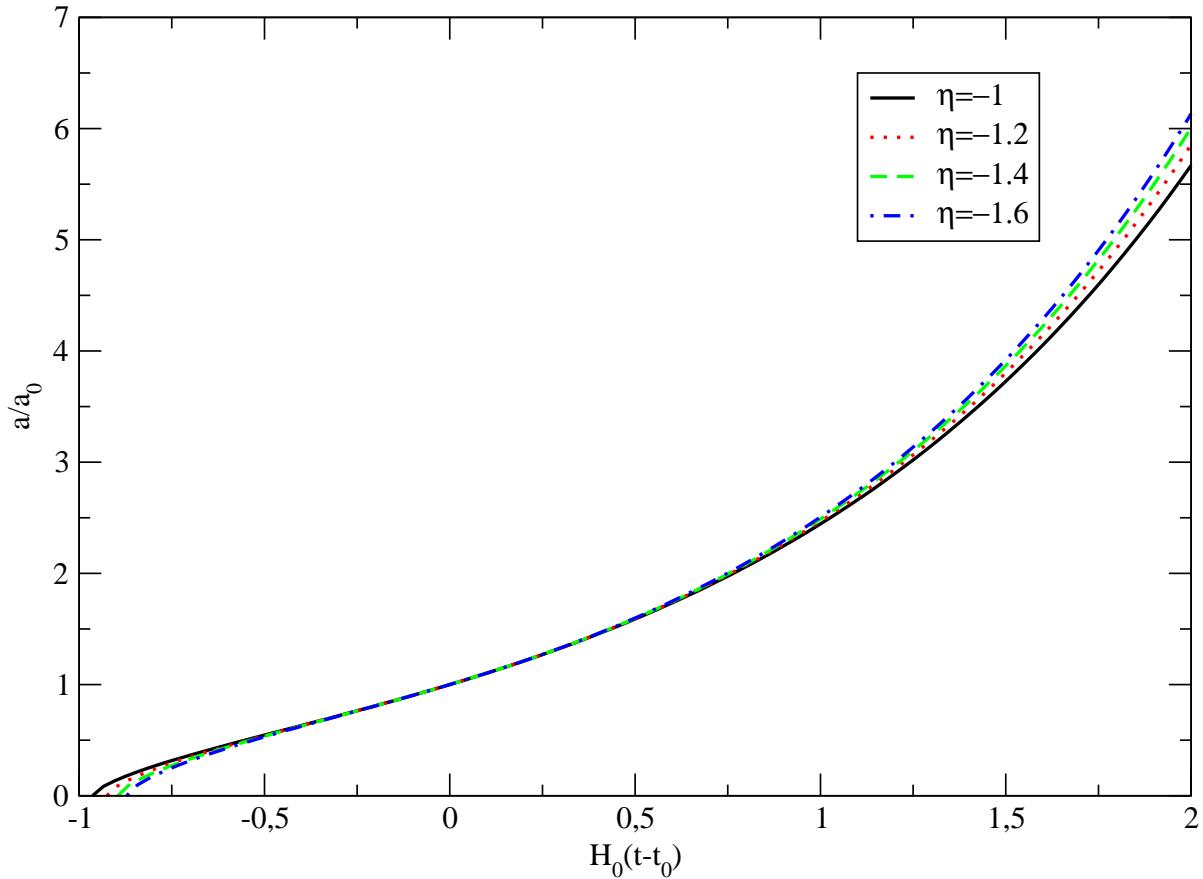


Figure 1: The evolution of the scale factor of the universe is shown for $\Omega_\Lambda^0 = 0.7$, $\gamma = 0$ and four different values of the parameter η . The difference between various models is the most prominent in the distant past and the distant future. More negative values of η lead to the faster growth of the scale factor in the distant future.

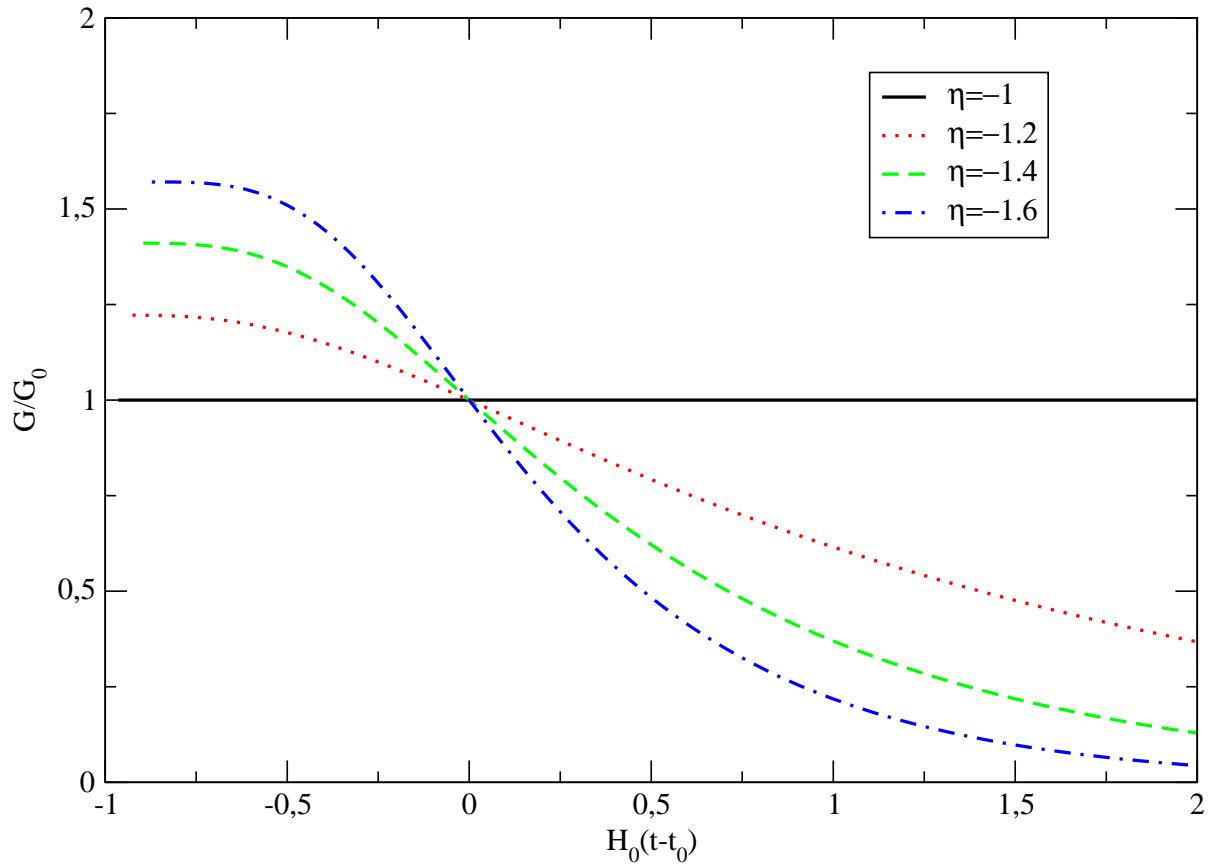


Figure 2: The evolution of the gravitational constant G is given for $\Omega_\Lambda^0 = 0.7$, $\gamma = 0$ and four various values of the parameter η . In the very early universe the change of G is very slow, while in the distant future G has a tendency towards 0. For more negative values of the parameter η , the quantity G changes faster.

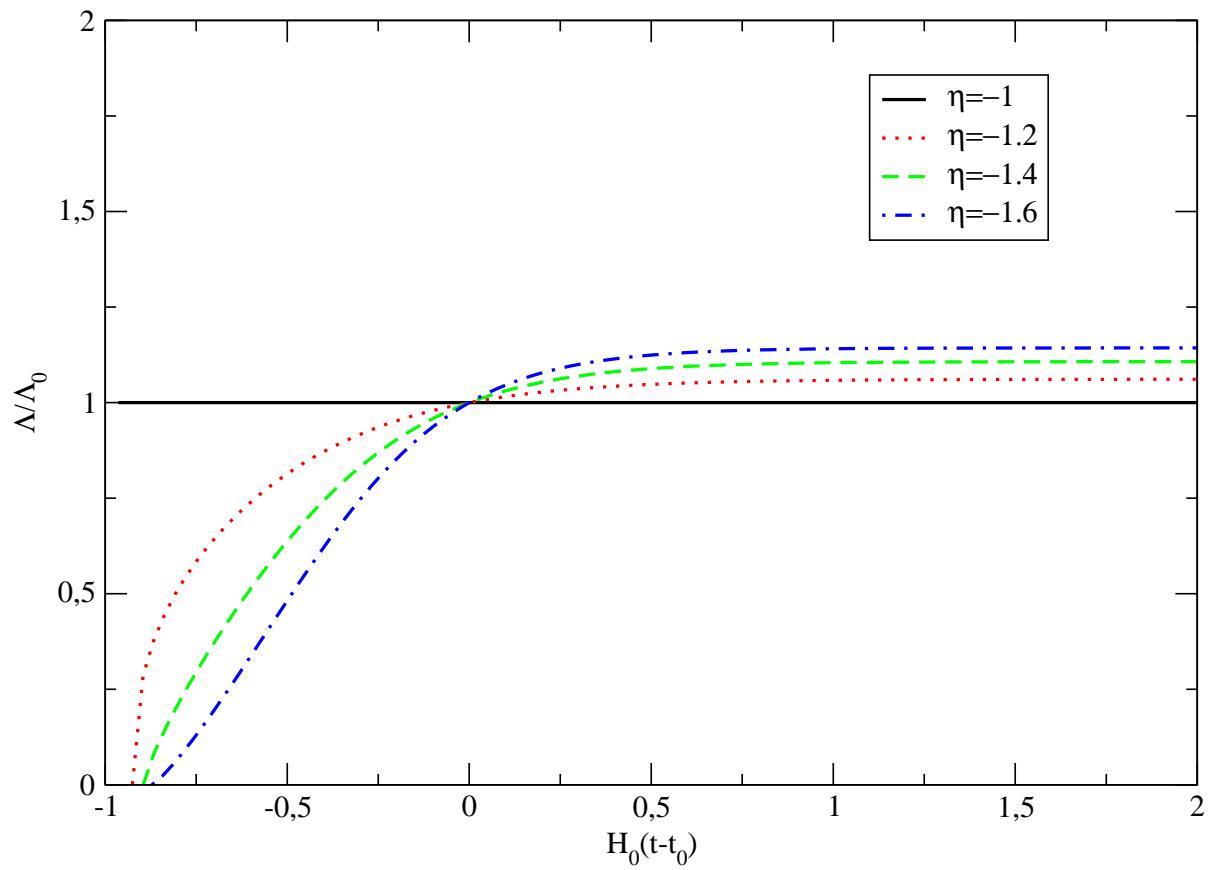


Figure 3: The change in time of the CC is depicted for $\Omega_\Lambda^0 = 0.7$, $\gamma = 0$ and four different values of the parameter η . In the distant future the quantity Λ reaches a saturation value. The onset of saturation is sooner for the smaller values of the parameter η .

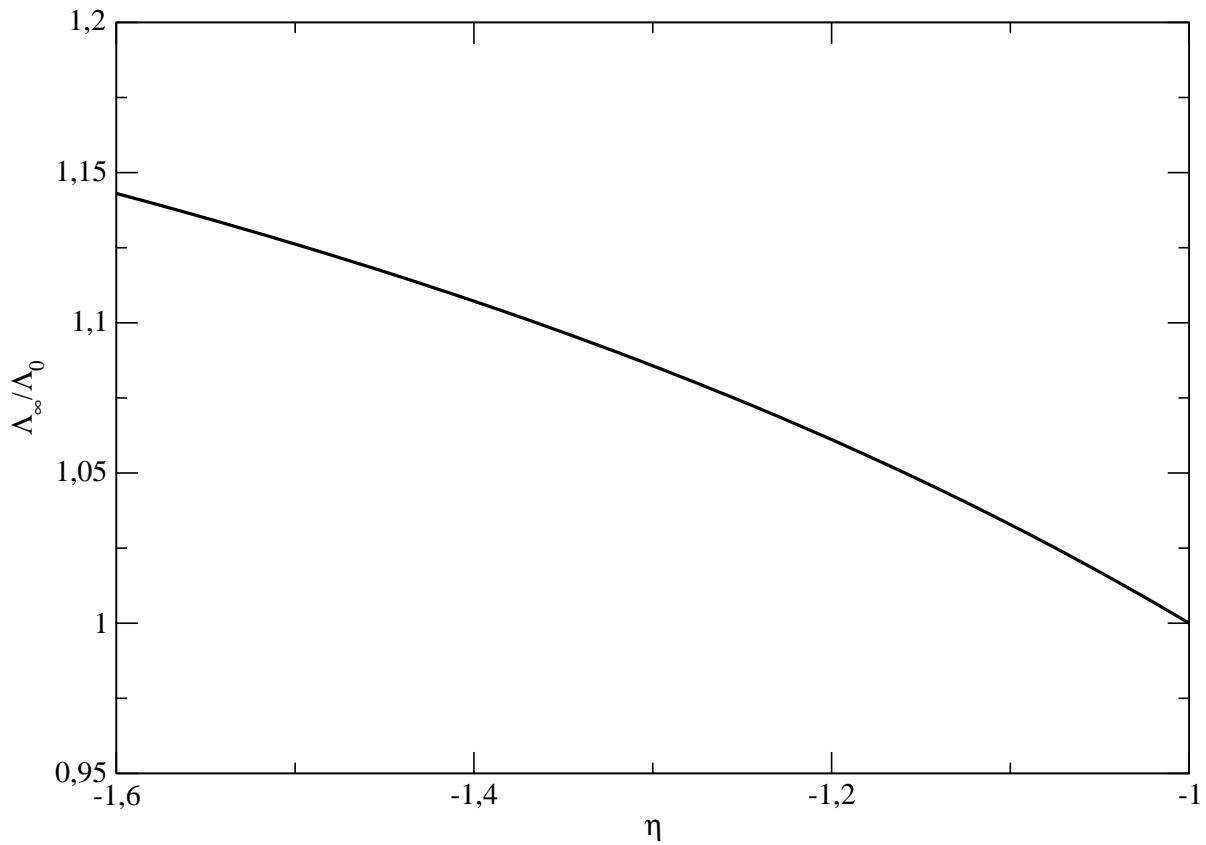


Figure 4: The asymptotic value Λ_∞ of the CC as a function of the parameter η . Parameters used in the calculation are $\Omega_\Lambda^0 = 0.7$, $\gamma = 0$. For a displayed range of values of the parameter η , the asymptotic value Λ_∞ is of the same order of magnitude as the value of the CC today, Λ_0 .